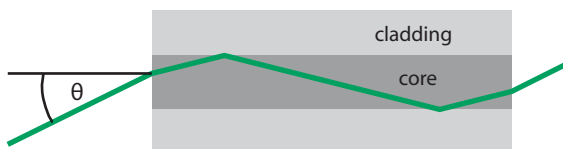


Principle of Waveguiding

Optical fibers represent a special kind of optical waveguide. A waveguide is a material structure that can “guide” light, i.e., let it propagate while preventing its expansion in one or two dimensions. Fibers are waveguides that guide in two dimensions and can effectively be used as flexible pipes for light.

In the simplest and most common case, the waveguide effect is achieved by using a **fiber core** with a refractive index that is slightly higher than that of the surrounding **cladding**. We initially consider **step-index** fibers, where the refractive index is constant within the fiber core (p. 8).



For a step-index fiber, the waveguide effect is often explained as resulting from total internal reflection of light rays at the core–cladding interface (see the figure). One easily comes to the conclusion that total internal reflection at the interface occurs if the external beam angle θ (in air) fulfills the condition

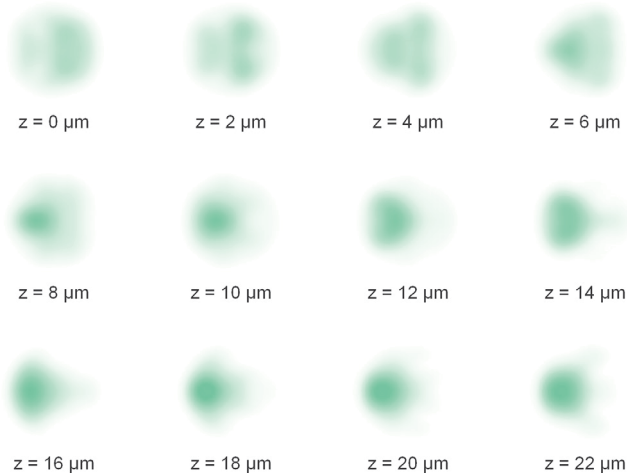
$$\sin \theta < NA \equiv \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2},$$

where NA is called the **numerical aperture** of the fiber (see p. 11). The fiber can then guide all light impinging the input face with angles fulfilling this condition, which does not depend on the core size.

This purely geometric picture gives reasonable results for large cores (strongly multimode fibers, see p. 14), but is invalid for small single-mode cores (p. 9), where the wave nature of light cannot be ignored.

Wave Propagation in Fibers

A precise description of light propagation in fibers requires the treatment of light as a wave phenomenon. A common method for numerical calculations is the **beam propagation method**. Starting with a certain electric field distribution at an input face, one calculates how the field propagates a small distance into the fiber. By applying further propagation steps, one can calculate the field distribution (and intensity distribution) everywhere in the fiber. In general, the field distribution can undergo sophisticated changes during propagation. The figure below shows an example for a multimode fiber (p. 14).



A very useful concept is that of **modes**. These represent field distributions with the special property that their shape in the transverse direction remains constant during propagation. These field distributions have only a phase change (which is the **propagation constant** β times the propagation distance z), and a change of power proportional to $\exp(-\alpha z)$ where α is the loss coefficient. (Note: the effective loss coefficient may be negative in an amplifying fiber—see p. 74.) In general, each mode can have its own values of α and β . The number of modes, their field distributions, and their α and β values depend on the optical frequency ν or the wavelength λ .

Calculation of Fiber Modes

Here we briefly consider how fiber modes are calculated in cases where two simplifying assumptions apply: the index contrast is small, and the refractive index depends only on the radius r (the distance to the fiber axis), but not the azimuthal angle φ . This excludes, e.g., fibers with an elliptical core.

If a field distribution $E(r, \varphi, z)$ corresponds to a mode, it must have a simple z dependence, leaving the shape of the intensity profile constant:

$$E(r, \varphi, z) = \Psi(r, \varphi) \exp(i\beta z)$$

with the **propagation constant** β . Absorption losses have been ignored here. The transverse field function $\Psi(r, \varphi)$ is further decomposed:

$$\Psi(r, \varphi) = F_{lm}(r) \cos(l\varphi),$$

where l describes the azimuthal dependence and must be an integer, as the field must stay unchanged for an increase of φ by 2π . Another solution contains $\sin(l\varphi)$ instead of $\cos(l\varphi)$. The second integer index m is necessary, as multiple solutions can exist for a given value of l .

By inserting the latter equation into the wave equation, one obtains the equation

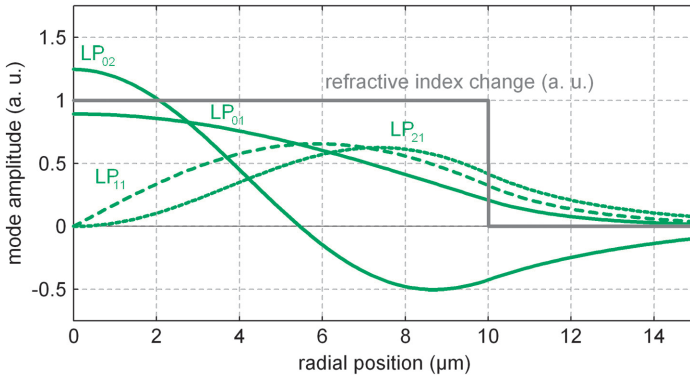
$$F_{lm}''(r) + \frac{F_{lm}'(r)}{r} + \left(n^2(r) k^2 - \frac{l^2}{r^2} - \beta_{lm}^2 \right) F_{lm}(r) = 0,$$

where $k = 2\pi/\lambda$ and the primes indicate derivatives with respect to r . This differential equation combined with suitable boundary conditions (e.g., $F \rightarrow 0$ for $r \rightarrow \infty$ in the case of guided modes, see p. 6) must be solved with analytical or numerical means.

Calculation of Fiber Modes (cont.)

For arbitrary values of β , the solutions for F will usually diverge for $r \rightarrow \infty$ and thus cannot represent guided-fiber modes. For some given (not too high) non-negative integer value of l and suitably chosen β values, however, one may find solutions that asymptotically go to zero for increasing r . The one with the highest value of β can be labeled with $m = 1$, and solutions with lower β obtain higher integer values of m . Numerical methods may be used for fibers with arbitrary index profiles.

As an example, the figure below shows the radial amplitude distributions of all of the solutions for a step-index fiber at a given wavelength.



For example, LP_{02} means that $l = 0$ (i.e., there is no φ dependence) and $m = 2$. The fundamental mode LP_{01} is closest to a simple Gaussian profile, extending somewhat beyond the core.

More sophisticated calculations are required for fibers with high index contrast or with an azimuthal dependence of the refractive index.¹⁸ Particularly difficult to calculate are modes of photonic crystal fibers (page 55), containing air holes.

Decomposition into Modes

Once all modes of a fiber are known, the propagation of a monochromatic beam with arbitrary field distribution along the fiber can be calculated in an efficient way:

- Decompose the initial field distribution $E_0(x,y)$ into fiber modes, i.e., consider it as a linear combination of modes:

$$E_0(x, y) = \sum_{l,m} a_{lm}(0) E_{lm}(x, y)$$

where $E_{lm}(x,y)$ is the field distribution for mode indices l and m . Calculate the initial complex amplitude coefficient $a_{lm}(0)$ of each mode using an overlap integral. Assuming normalized mode functions, this reads:

$$a_{lm}(0) = \int E_{lm}^*(x, y) E_0(x, y) dx dy.$$

It is often sufficient to consider only guided modes (p. 6), since cladding modes (p. 7) are usually fairly lossy and thus do not contribute to the output.

- For all considered modes, calculate the change of amplitude and phase during propagation using the known α and β values:

$$a_j(z) = a_j(0) \exp\left(-\frac{\alpha_j}{2}z + i\beta_j z\right).$$

- Calculate the final field distribution based on the final mode coefficients and the mode fields:

$$E(x, y, z) = \sum_{l,m} a_{lm}(z) E_{lm}(x, y).$$

For polychromatic beams, the different frequency components must be propagated separately.

Types of Fiber Modes

Fibers can support different types of modes:

- **Guided modes** are those with intensity distributions limited to the core and its immediate vicinity. Their field distributions decay exponentially in the cladding. Guided modes normally exhibit rather small propagation losses.
- In some situations, so-called **leaky modes** occur, which are concentrated around the core but lose some power into the cladding.
- **Cladding modes** (p. 7) have intensity distributions that essentially fill the full cladding region, thus also reaching the outer surface of the cladding, where they often experience large power losses. The intensity in the fiber core is substantial for some cladding modes, but very small for others.

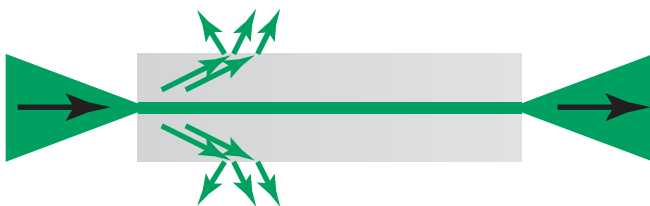
The number of guided modes depends strongly on the fiber design:

- Fibers with only a single guided mode per polarization direction are called **single-mode fibers** (p. 9). Single-mode guidance is usually restricted to some wavelength range. Typically, there is multimode guidance for wavelengths below some cut-off wavelength, whereas the propagation losses increase for longer wavelengths.
- Fibers with more than one guided mode are **multimode fibers** (p. 14). Some support just a few guided modes, others may support a very large number.

In general, the number of guided modes increases with decreasing wavelength.

Cladding Modes

Cladding modes are propagation modes of a fiber (or other waveguide) that are not confined to the surroundings of the core. When trying to launch light into the fiber core, one may inject some part of the power into cladding modes, if the input light is not well matched to the guided mode(s).



The fiber cladding is often surrounded by a polymer coating, which not only mechanically protects the fiber, but also causes high propagation losses for cladding modes. Any power in cladding modes may then quickly decay and will not get to the fiber end (except when the fiber is rather short). The light launched into the core has much lower propagation losses, so that its power remains nearly constant, unless the fiber is very long.

The high loss for cladding modes is convenient, e.g., when checking how efficiently light is launched into the fiber core, or when measuring the propagation losses of the fiber core.

Residual light in cladding modes can be disturbing, e.g., when one tries to measure strong absorption in a short, highly doped rare-earth-doped fiber. Here, light in cladding modes evades the absorption (because the cladding is undoped) and thus simulates a weaker degree of core absorption. Elimination of light in cladding modes may then be accomplished by splicing a longer undoped single-mode fiber to the test fiber. Another possibility is to use a droplet of index-matching fluid on the fiber where its coating is stripped off.

Step-Index Fibers

Step-index fibers are optical fibers with the simplest possible refractive index profile: a constant refractive index n_{core} in the core with some radius a , and another constant value n_{cladding} in the cladding. We always have $n_{\text{core}} > n_{\text{cladding}}$, as otherwise no guided modes exist.

Some important parameters of step-index fibers are:

- The **numerical aperture**

$$NA \equiv \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2}$$

(as already defined on p. 1) is related to the refractive index contrast. If the index difference $\delta n = n_{\text{core}} - n_{\text{cladding}}$ is small (which is usually the case), we can approximate

$$NA \approx \sqrt{2 n_{\text{cladding}} \delta n} .$$

- The **V number** (p. 10) is

$$V = \frac{2\pi}{\lambda} a NA = \frac{2\pi}{\lambda} a \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2} .$$

The V number determines the number of guided modes. For example, single-mode propagation is obtained for V smaller than ≈ 2.405 . Also, the V number determines the fraction of the power of a mode that is propagating within the core. Obviously, the V number depends on the wavelength, whereas that dependence is very weak for the numerical aperture.

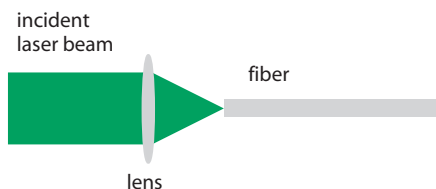
Note that these parameters are not universally defined for fibers with other than step-index profiles.

Single-Mode Fibers

In some wavelength regions, a fiber with a small core may have only a single guided mode per polarization direction. In that regime, the intensity profile at the fiber output has a fixed shape, independent of the launch conditions and the spatial properties of the injected light, provided that no cladding modes (p. 7) can carry significant power to the fiber end. The launch conditions do, however, influence the efficiency with which light can be coupled into the guided mode. Efficient launching requires that the light on the input fiber end has a complex amplitude profile similar to that of the guided mode. That implies

- that the injected light must have a high beam quality;
- that this light is focused on the input fiber end in such a way that a spot with the proper size and position is obtained on the fiber end, and that the beam direction is aligned correctly.

If these conditions are not fulfilled, a large fraction of the power of incident light gets into cladding modes.



A long-term, stable, efficient launch of a free-space laser beam into a single-mode fiber requires a stable opto-mechanical setup containing a focusing lens with appropriate focal length (depending on the fiber's mode size and the initial beam size) and a holder for the fiber end. These parts need to be aligned precisely, and stably fixed without excessive thermally induced drifts.
