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Abstract. The workshop Measurement Positioning System (wMPS) is a large-scale measurement system that better copes with the current challenges of dimensional metrology. However, as a distributed measuring system with multiple transmitters forming a spatial measurement network, the network topology of transmitters relative to the receiver exerts a significant influence on the measurement accuracy albeit one that is difficult to quantify. An evaluation metric, termed the geometric dilution of precision (GDOP), is introduced to quantify the quality of the network topology of the wMPS. The GDOP is derived from the measurement error model of wMPS and its mathematical derivation is expounded. Two significant factors (density and layout of the transmitter) affecting the network topology are analyzed by simulations and experiments. The experimental results show that GDOP is approximately proportional to the measurement error. More transmitters, and a relatively good layout thereof, can decrease the value of GDOP and the measurement error. © *The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI.* [DOI: 10.1117/1.OE.56.9.094101]

1 Introduction

Large-scale metrology[1–3] plays a vital role in industrial manufacturing and quality control. Today, large-scale metrology is challenged by many more demanding requirements:[3,4] higher accuracy, more efficient measurement, and the need to adapt to a more complex environment. A centralized measurement system,[5] such as a single laser tracker, has significant disadvantages in a complex environment, where its portability is not ideal and there are many line-of-sight problems. Compared with a centralized system, a distributed measurement system[5–7] with multiple measuring stations shows potential advantages when used in manufacturing industries because of its scalable measurement and concurrent measurement capability.

Among those distributed measurement systems, rotary-laser measurement systems, such as the workshop Measurement Positioning System (wMPS)[6] and the indoor Global Positioning System (iGPS),[7] have developed rapidly. As a nonorthogonal system with multiple transmitters forming a network, the measurement results are calculated by triangulation using the information collected from transmitters and the measurement accuracy is related to the topological structure of the transmitters relative to the receiver.

The importance of network topology of rotary-laser systems has been recognized in previous work: Schmitt et al.[8] evaluate the performance of iGPS with different layouts based on the Monte-Carlo simulation; however, the Monte-Carlo simulation needs a suitable stochastic model for the iGPS, which is both complex and uncertain, and they analyze just one factor affecting the measurement uncertainty. Maisano et al.[5] introduce various factors affecting the measurement and performance of the iGPS, which depends on its physical configuration; however, their work lacks experimental evidence with which to verify the relationship between network topology and measurement accuracy. None of the aforementioned analyses uses the law of error propagation to show how the network topology directly influences the measurement accuracy, and there were no quantitative criteria to evaluate the effect of network topology on measurement accuracy.

In this paper, we introduce a popular dimensionless metric, termed the geometric dilution of precision (GDOP), to quantify the effect of the network topology on the final measurement accuracy and we analyze two significant factors affecting this metric. The GDOP is calculated by using a mathematical model based on the rotary-laser measurement system, which is known as wMPS. Two significant factors affecting the GDOP are analyzed by simulations and experiments in detail. wMPS with different network topologies has different values of GDOP. A lower value of GDOP represents a better network topology of transmitters relative to the receiver, and it results in a more accurate measurement.

The rest of this manuscript is organized as follows: in Sec. 2, the GDOP is introduced and its derivation, based on the mathematical model of wMPS, is presented; Sec. 3 presents an analysis of two significant factors that influence the GDOP by simulations; in Sec. 4, based on the existing platform, verification experiments are designed; and in Sec. 5, the conclusions are presented along with a brief overview for potential future research.

2 Geometric Dilution of Precision and Related Calculation

2.1 *Introduction of Geometric Dilution of Precision*

From the ideal measurement model of wMPS, the signals from the transmitters intersect at one point; however, each

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transmitter is subject to an angular measurement error, which results in an error in the final coordinate calculation. As shown in Fig. 1, the angular measurement error of the transmitter is α and the intersection part of signals is an oddly shaped area. The smaller the area of the shaded part, the lower the coordinate measurement error.

In Fig. 1(a), the transmitters are so close that the area of the shaded part is relatively large, and so, the measurement accuracy is relatively low. In Fig. 1(b), the transmitters are deployed far apart. Even though the angular measurement error of transmitters remains the same as in Fig. 1(a), the area of the shaded part is smaller, so, the measurement accuracy is relatively high. In Fig. 1(c), three transmitters are deployed and the area of the shaded part is the smallest of these cases. So, the measurement accuracy is the highest. These two-dimensional schematic diagrams explain how the network topology influences the final measurement accuracy.

In the Global Navigation Satellite System, the concept of dilution of precision (DOP) has been used as an accuracy metric to evaluate the geometric effect of satellite configurations on GPS accuracy.[9,10] However, as a distributed measurement system with the principle of angle intersection, the calculation of the GDOP of wMPS is different from that used in the GPS with the principle of multilateration. So, to quantify the quality of the network topology of the wMPS, GDOP is introduced and calculated.

2.2 Measurement Model of wMPS

The measurement principle of the transmitter is illustrated in Fig. 2. There are two laser modules, which can be considered as two nonparallel planes rotating around the rotation axis. These two laser planes are called "plane 1" and "plane 2," respectively. In Fig. 2(a), the transmitter coordinate system is defined. The optical axis of a laser device at the initial time is defined as axis x and the rotation axis is defined as axis z. According to the right-hand rule, axis y is defined. Plane 1 and plane 2 are installed at a 90 deg angle, which is called "λ" in Fig. 2(b). When the transmitter works, two laser planes pass through the receiver successively. These optical scanning signals are converted into the scanning angles of each plane, namely θ_1 and θ_2. The horizontal angle α and

vertical angle β of the transmitter relative to the receiver are determined by these two scanning angles.

According to the measurement principle of wMPS,[11,12] the observation equations are given by

$$F_{mn}(\theta_{mn}, C) = [a_{mn}\ b_{mn}\ c_{mn}\ d_{mn}]$$
$$\times \begin{bmatrix} R(\theta_{mn})^T & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_m & T_m \\ 0 & 1 \end{bmatrix} [x\ y\ z\ 1]^T = 0, \quad (1)$$

where $[a_{mn}\ b_{mn}\ c_{mn}\ d_{mn}]$ denotes the internal parameters of transmitters, $m(m \geq 2)$ denotes the number of transmitters, and $n(n = 1, 2)$ denotes the number of laser planes of each transmitter; $R(\theta_{mn})^T$ refers to the rotation matrix of the laser planes of transmitter and θ_{mn} refers to the scanning angle that the planes rotate around the rotation axis of the transmitter; R_m and T_m represent the rotation and translation matrix, which relate the transmitter coordinate system to the measurement coordinate system; and $C = [x\ y\ z]$ stands for the coordinate of the receiver in the measurement system.

In Eq. (1), scanning angle θ_{mn} is the observed value and the coordinate $[x\ y\ z]$ is the measurand. These equations are implicit functions. On the basis of the Taylor series expansion algorithm of implicit function[13] around the initial coordinate of receiver $[x_0\ y_0\ z_0]$ and neglecting the higher order terms, Eq. (1) can be expressed as follows:

$$F(\theta, C) = F(\theta_0, C_0) + \frac{\partial F(\theta, C)}{\partial \theta}(\theta - \theta_0)$$
$$+ \frac{\partial F(\theta, C)}{\partial x}(x - x_0) + \frac{\partial F(\theta, C)}{\partial y}(y - y_0)$$
$$+ \frac{\partial F(\theta, C)}{\partial z}(z - z_0). \quad (2)$$

If $\frac{\partial F(\theta, C)}{\partial \theta} \neq 0$, Eq. (2) can be rewritten as follows:

$$\Delta\theta = G \cdot \Delta C, \quad (3)$$

where $\Delta\theta = (\theta - \theta_0)$, $G = -[\frac{\partial F(\theta, C)}{\partial x}, \frac{\partial F(\theta, C)}{\partial y}, \frac{\partial F(\theta, C)}{\partial z}]/\frac{\partial F(\theta, C)}{\partial \theta}$, $\Delta C = [x - x_0, y - y_0, z - z_0]'$.
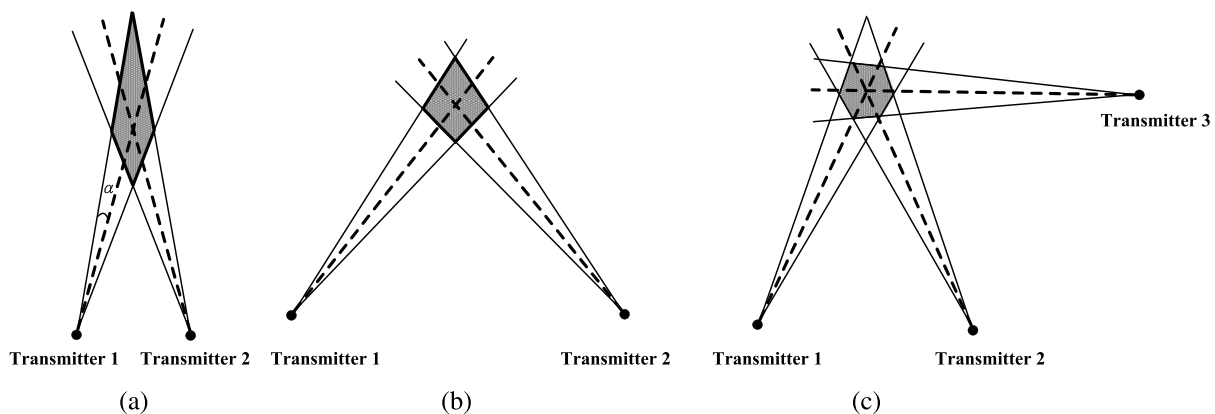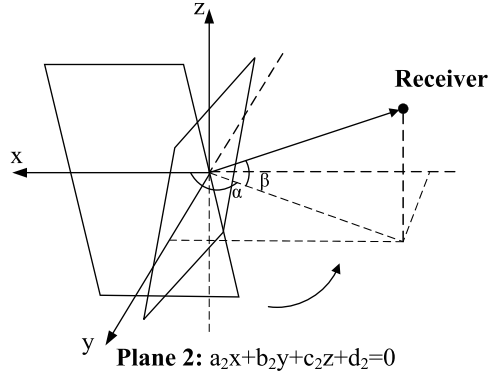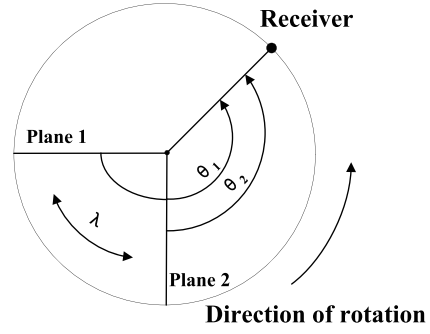

Fig. 1 Network topology influences measurement accuracy: (a) two transmitters with a relatively low measurement accuracy, (b) two transmitters with a relatively high measurement accuracy, and (c) three transmitters with the highest measurement accuracy.

Plane 1: $a_1x+b_1y+c_1z+d_1=0$

Plane 2: $a_2x+b_2y+c_2z+d_2=0$

(a)

(b)

Fig. 2 Mathematical model and measurement schematic of transmitter: (a) mathematical model of transmitter and (b) scanning angle measurement.

Considering different transmitters with different observation errors, it is necessary to add the weight matrix into Eq. (3) as follows:

$$W\Delta\theta = WG \cdot \Delta C, \qquad (4)$$

where $W = \begin{bmatrix} w_1^2 & & & \\ & w_2^2 & & \\ & & \ddots & \\ & & & w_n^2 \end{bmatrix}$ and w_n represents the weight factor.

The weight factor is obtained as the inverse of the variance of the observation error, which is associated with the distance between the receiver and the transmitter. If the coordinate of the receiver is unknown, the weight factors are set as the same value. After the coordinate is calculated, the weight factors should be set as the value, which is inversely proportional to the distance to optimize the coordinate.

Equation (4) above is a linear equation and can be obtained by the least square algorithm. So, L is introduced to define the residual sum of squares as follows:

$$L(\Delta C) = \|WG \cdot \Delta C - W\Delta\theta\|^2 = \min. \qquad (5)$$

There exists ΔC to minimize the equation $\|WG \cdot \Delta C - W\Delta\theta\|^2$, and we can get

$$\Delta C = (G^T W^T W G)^{-1} G^T W^T W \Delta\theta. \qquad (6)$$

In Eq. (3), the relationship between the observations $\Delta\theta$ and the measurand ΔC is formulated without measurement errors, however, the noise of observations is inevitable. Therefore, Eq. (3) is rewritten as follows:

$$\Delta\theta + \varepsilon_\theta^{(mn)} = G \cdot \begin{bmatrix} \Delta x + \varepsilon_x \\ \Delta y + \varepsilon_y \\ \Delta z + \varepsilon_z \end{bmatrix}, \qquad (7)$$

where $\varepsilon_\theta^{(mn)}$ represents the observation error and $\varepsilon_x, \varepsilon_y, \varepsilon_z$ represents the positioning error in x, y, z directions, respectively, m ($m \geq 2$) denotes the number of transmitters, and n

($n = 1, 2$) denotes the number of laser planes of each transmitter.

Substituting the $\Delta\theta$ of Eq. (3) into Eq. (7), the relationship between the positioning error and the observation error is as follows:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = (G^T G)^{-1} G^T \varepsilon_\theta^{(mn)}. \qquad (8)$$

To simplify the error model, there are two hypotheses: first, observation errors of different transmitters are independent and two laser planes in the same transmitter are uncorrelated. Second, observation errors of each laser plane in each transmitter have the same Gaussian distribution with the same mean value and variance. The mean value of the observation error is zero, and the variance of the observation error is σ_0^2:

$$E\left[\varepsilon_\theta^{(mn)}\right] = 0, \quad V\left[\varepsilon_\theta^{(mn)}\right] = \sigma_0^2. \qquad (9)$$

Therefore, the covariance matrix of observation errors is calculated as follows:

$$\begin{aligned} \text{cov}(d\theta) &= E(\{\varepsilon_\theta^{(mn)} - E[\varepsilon_\theta^{(mn)}]\}\{\varepsilon_\theta^{(mn)} - E[\varepsilon_\theta^{(mn)}]\}^T) \\ &= \begin{bmatrix} \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \sigma_0^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_0^2 \end{bmatrix} = \sigma_0^2 I, \end{aligned} \qquad (10)$$

where I is the unit matrix.

According to the covariance propagation,[14,15] the covariance of dC can be expressed as

$$\begin{aligned} \text{cov}(dC) &= E[dC \cdot dC^T] = E[(\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z)^T (\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z)] \\ &= H\text{cov}(d\theta)H^T, \end{aligned} \qquad (11)$$

where $H = (G^T G)^{-1} G^T$.

Suppose that two transmitters are involved in the system and $H = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_5 & h_6 & h_7 & h_8 \\ h_9 & h_{10} & h_{11} & h_{12} \end{bmatrix}$, Eq. (8) can be rewritten as follows:

$$
\begin{bmatrix} \sigma_x^2 & & \\ & \sigma_y^2 & \\ & & \sigma_z^2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_5 & h_6 & h_7 & h_8 \\ h_9 & h_{10} & h_{11} & h_{12} \end{bmatrix}
$$
$$
\times \begin{bmatrix} \sigma_0^2 & & & \\ & \sigma_0^2 & & \\ & & \sigma_0^2 & \\ & & & \sigma_0^2 \end{bmatrix} \begin{bmatrix} h_1 & h_5 & h_9 \\ h_2 & h_6 & h_{10} \\ h_3 & h_7 & h_{11} \\ h_4 & h_8 & h_{12} \end{bmatrix}, \tag{12}
$$

where $\sigma_x^2, \sigma_y^2, \sigma_z^2$ is the variance of positioning error in x, y, z directions, respectively.

So, the variances of the measurands, respectively, correspond to:

$$
\sigma_x^2 = (h_1^2 + h_2^2 + h_3^2 + h_4^2)\sigma_0^2,
$$
$$
\sigma_y^2 = (h_5^2 + h_6^2 + h_7^2 + h_8^2)\sigma_0^2,
$$
$$
\sigma_z^2 = (h_9^2 + h_{10}^2 + h_{11}^2 + h_{12}^2)\sigma_0^2. \tag{13}
$$

From Eq. (11) above, the errors of the measurand (coordinate of receiver) are affected by two factors:

The transfer coefficient matrix: HH^T.

The covariance of observation error (or the angular measurement error): $\text{cov}(d\theta)$.

2.3 Calculation of GDOP

Just as discussed above, the transfer coefficient matrix HH^T shows how the geometric distribution of transmitters affects the final measurement accuracy. The DOPs are defined from the coefficient matrix.

The matrix HH^T can also be described as

$$
HH^T = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}. \tag{14}
$$

And we can obtain

$$
\sigma_x^2 = h_{11}\sigma_0^2, \quad \sigma_y^2 = h_{22}\sigma_0^2, \quad \sigma_z^2 = h_{33}\sigma_0^2. \tag{15}
$$

Above all, the standard deviation of the two-dimensional horizontal positioning error is described as follows:

$$
\sigma_h = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{h_{11} + h_{22}}\sigma_0. \tag{16}
$$

The standard deviation of the vertical positioning error is described as follows:

$$
\sigma_v = \sqrt{\sigma_z^2} = \sqrt{h_{33}}\sigma_0. \tag{17}
$$

The standard deviation of the three-dimensional (3-D) positioning error is described as follows:

$$
\sigma_p = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = \sqrt{h_{11} + h_{22} + h_{33}}\sigma_0. \tag{18}
$$

The HDOP, VDOP, and GDOP are the horizontal DOP, the vertical DOP, and the geometry DOP, respectively. These DOPs of wMPS are defined as follows:

$$
\text{HDOP} = \sqrt{h_{11} + h_{22}}, \text{VDOP} = \sqrt{h_{33}}, \text{GDOP}
$$
$$
= \sqrt{h_{11} + h_{22} + h_{33}}. \tag{19}
$$

These DOPs show the effect of network topology on measurement accuracy, and the GDOP is a scaling factor with no unit, which amplifies the angular error of wMPS. The lower the GDOP, the lower the measurement error.

3 Factors Affecting the GDOP of the Measurement Network

It is worth remarking, with reference to Fig. 1, that two significant factors related to the network topology influence the value of GDOP:

1. Layout of transmitters relative to the receiver.
2. Density of network transmitters.

These two factors are analyzed below.

3.1 Layout of Transmitters

As mentioned in Fig. 1, the area of intersection has a positive correlation with the measurement error, and different layouts of transmitters result in different areas of intersection.

A series of simulations are carried out to show how the layout of transmitters influences the GDOP. Four transmitters are deployed in an area about 15 m × 15 m. Because the working distance of the transmitter is about 4 to 20 m, the distance between the simulation area and the transmitter should be more than 4 m and less than 20 m. In the meantime, because of the need for the intersection angle to be neither too large nor too small, the transmitters are placed more than 5 m from each other in these simulations. The simulation part lies in the horizontal plane with the area from 5 m to 10 m in the x-direction and that from 4 m to 10 m in the y-direction. The transmitters are deployed in "line," "C," and "square" types, respectively. The GDOP of each point in the simulation part is calculated using the mathematical model above.

As shown in Fig. 3, different layouts of transmitters and the receiver generate different distributions of GDOP. If transmitters are deployed in "line" type, the area near the transmitters has a lower GDOP and the area far from the transmitters has a higher GDOP. When transmitters are deployed in "C" type, the middle part (where all the transmitters can be seen) has a lower GDOP and the area where only two transmitters can be seen (such as the origin) has a higher GDOP. When transmitters are deployed in "square" type, all of the simulation area can be seen by each of the transmitters. So, the middle part (where the intersection angle is near-optimal) has a lower GDOP than that where the intersection conditions are poor. Above all, wMPS with a "C" type deployment is better than wMPS with the other two forms of deployment.
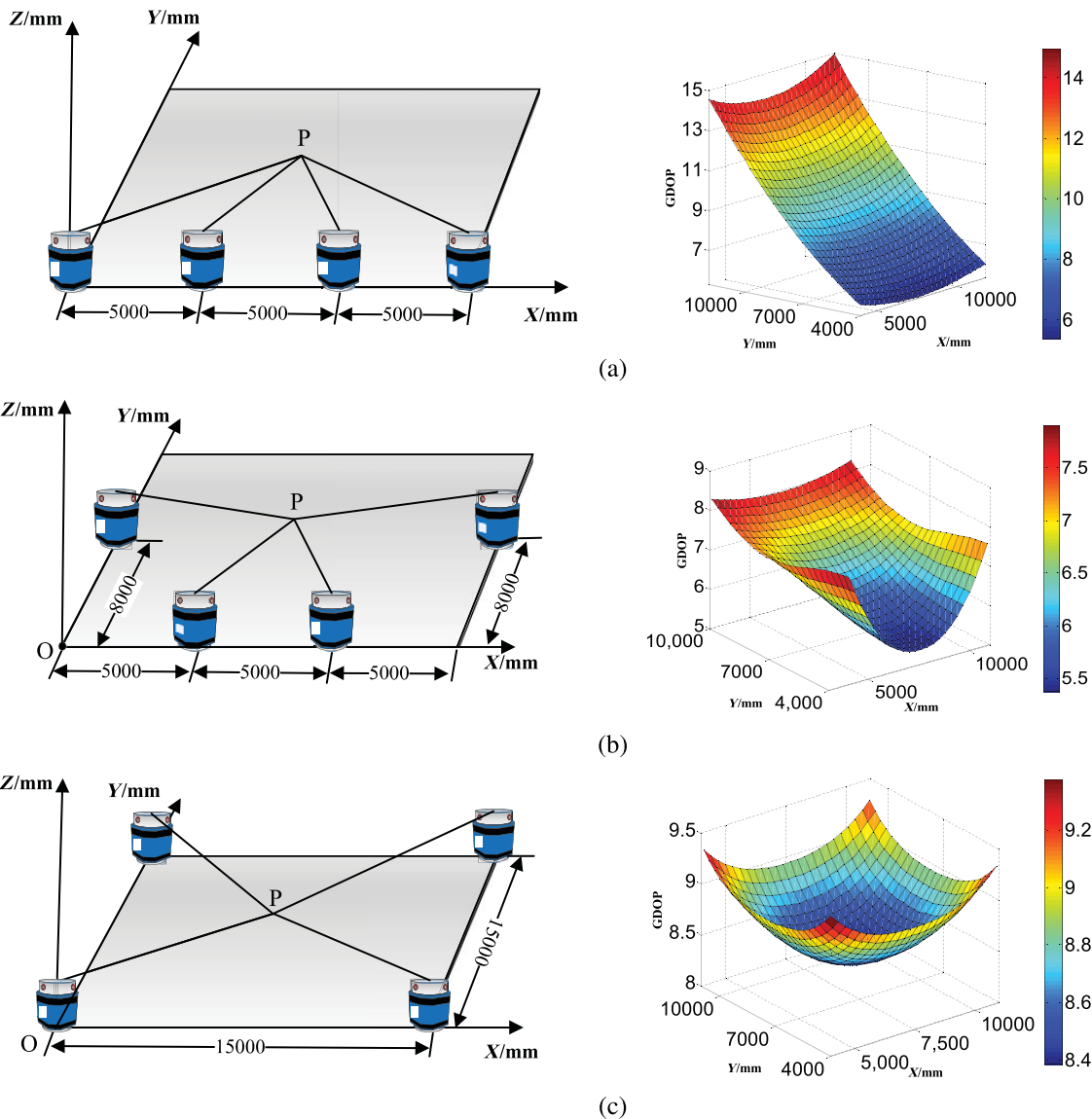
Fig. 3 Different layouts of transmitters affecting the GDOP: (a) "line" type deployment, (b) "C" type deployment, and (c) "square" type deployment.

The simulation results prove that the layout of transmitters relative to the receiver influences the value of GDOP and a good layout can decrease the value of GDOP.

3.2 Density of Network Transmitters

Based on the mathematical principles, more transmitters bring in more redundant information so that the solution of the observation equations is more accurate.

To validate the relationship between density of transmitters and the DOPs, two simulations are carried out. In these simulations, the working area where the transmitters are deployed is about 15 m × 15 m and the simulated area to be measured by the transmitters is about 3 m × 3 m in the middle of the working area. The DOPs of each point in the simulations are calculated directly by the model above. The DOP and GDOP in Fig. 4 are the average values of all the points in the simulated area. In the first simulation, the density of transmitters changes from two to eight, and the

influence of the number of transmitters on the GDOP, HDOP, and VDOP is shown in Fig. 4(a). In the second simulation, the number of transmitters changes from three to eight and the influence thereof on the GDOP with different layouts is shown in Fig. 4(b).

The result from Fig. 4(a) shows that the values of DOPs (GDOP, HDOP, or VDOP) are the highest if wMPS works with two transmitters and its accuracy is the lowest. Three or four transmitters can improve the measurement accuracy quickly and the DOPs also decrease quickly. When using five transmitters, the DOPs decrease more slowly and the cost of the transmitters increases. With more than six transmitters, the improvement in accuracy is not obvious. From Fig. 4(b), the result shows that the GDOP decreases with the number of transmitters increasing with different layouts. Meanwhile, the GDOP with the "C" type deployment is lower than that of any other deployments. This proves that the layout of the transmitters also exerts a significant influence on the GDOP of wMPS too.
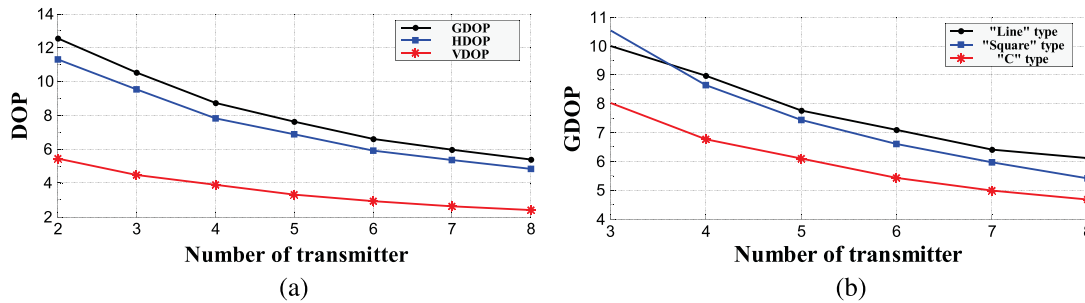
Fig. 4 DOP simulations: (a) relationship between the number of transmitters and GDOP, HDOP, and VDOP. (b) Relationship between the number of transmitters with different layouts and GDOP.

Above all, the density and layout of transmitters in the working area exert a significant influence on the GDOP, HDOP, or VDOP. The more transmitters involved in the wMPS, the lower the GDOP, and the more accurate the system.

4 Experiments

4.1 Experimental Platform Set-Up

To validate the relationship between the measurement accuracy and the GDOP as well as the network topology factors affecting the GDOP, verification experiments are designed based on the wMPS. In these experiments, the observation errors are supposed to be the same. To reduce the error caused by the environment, the wMPS is put in a relatively spacious area without obstruction, and the humidity and temperature remained within acceptable ranges of variation throughout.

As shown in Fig. 5, there are eight measurement points with magnetic nests deployed in a fixed position about 7 to 9 m away. The magnetic nest is a holder for 38.1-mm diameter sphere, in which is placed either a spherically mounted reflector of the laser tracker or a receiver of the wMPS. The receiver of the wMPS has the same size as the SMR of the laser tracker to facilitate comparison. wMPS is prepared to form a measurement network and this measurement network is calibrated by the scale bar of known length.[16]

The wMPS is deployed with different layouts and different densities of transmitters, respectively: two, three, and four transmitters (all in "line" type), and four transmitters (in "C" type) are tested. These eight points are measured by the laser tracker to form the reference measurement. About this laser tracker which is Leica AT901—LR, the maximum permissible error (MPE) of the angular accuracy is $\pm 15 \ \mu m + 6 \ \mu m/m$, and the MPE of interferometer accuracy is $\pm 0.4 \ \mu m + 0.3 \ \mu m/m$.

Meanwhile, these points are measured by the wMPS with these different network topologies, respectively.

4.2 Verification of Coordinate Measurement

In these experiments, the coordinate system of the wMPS with different types of network topologies is transformed into the laser tracker coordinate system by least-squares fitting.[17] In Fig. 6, the 3-D coordinate deviations between the laser tracker and the wMPS with different types of network topologies (two transmitters in line type, three transmitters in line type, four transmitters in line type, and four transmitters in C type) are shown. In addition, "Two in line," "three in line," and "four in line" refer to the coordinates measured by two, three, and four transmitters deployed in "line" type, respectively, "four in C" refers to the coordinate measured by four transmitters deployed in "C" type.
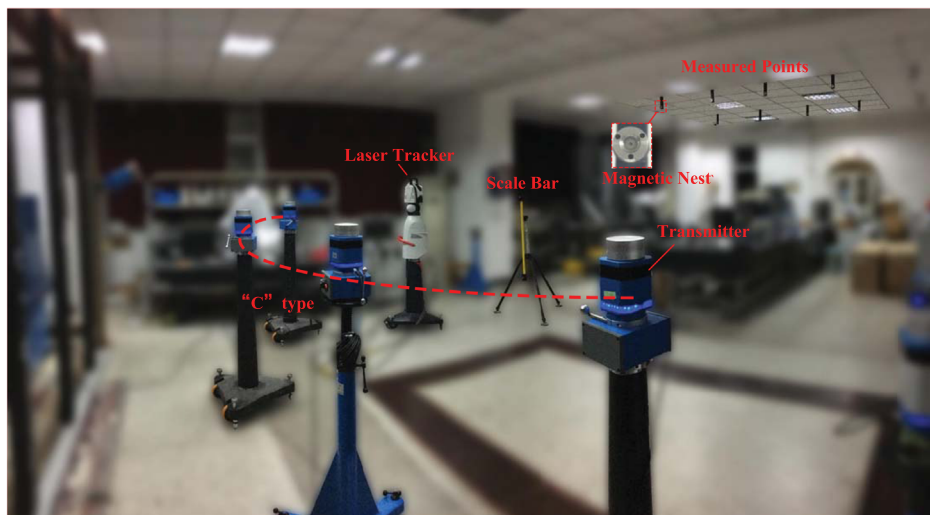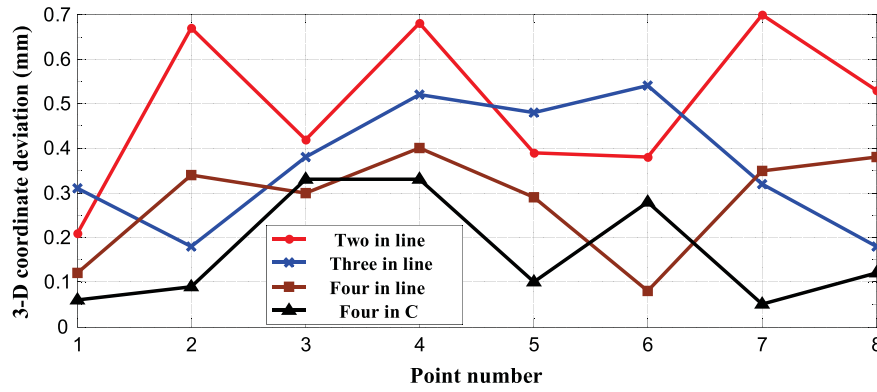

Fig. 5 Platform of experiments.

Fig. 6 Coordinate comparison between the laser tracker and the wMPS with different network topologies.

As shown in Fig. 6 above, measurement with two transmitters in "line" type has the largest deviation on average, and measurement with four transmitters in "C" type has the smallest deviation on average. Generally, more transmitters involved in the measurement with the same layout results in a more accurate measurement. Besides, with the same number of transmitters, a good layout also results in a more accurate measurement. For example, the coordinate deviation of wMPS with four transmitters in C type is smaller than that with four transmitters in line type on average. Because of the differences of each transmitter and the assembly error of the receiver, there are some exceptions as shown in Fig. 6.

To summarize these experiments above, the average GDOP of these eight points is calculated by models with different network topologies. The 3-D coordinate deviation of each comparison experiment and the average GDOPs with these four layouts are shown in Fig. 7.

In Fig. 7, the "L2," "L3," and "L4" represent the measurement with two, three, and four transmitters in line type, respectively, and the "C4" represents the measurement with four transmitters in C type. The relationship between GDOP and network topology is shown as a blue line, and the relationship between coordinate deviation and network topology is shown in red. These two lines have the same trend, which suggests that there is an approximately proportional relationship between the coordinate deviation of measured points and the value of GDOP. This relationship agrees with the
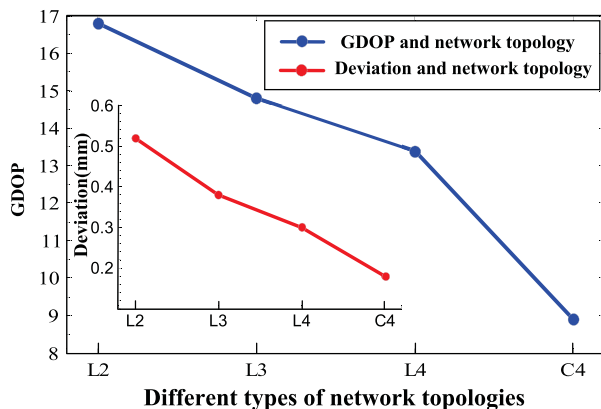
mathematical model above. So, the GDOP is able to be used as a quantitative criterion to evaluate the effect of network topology on the measurement accuracy. In the meantime, it can be concluded that the more transmitters involved in a measurement, the lower its GDOP and the more accurate the measurement. wMPS with four transmitters in the "C" type deployment has a better accuracy and lower GDOP than wMPS in the "line" type deployment. The density of transmitters and the layout of transmitters are two significant factors affecting the GDOP and the measurement accuracy.

These experiments prove that the GDOP is a feasible metric with which to evaluate the effect of network topology on measurement accuracy of wMPS. The experimental results agree well with the simulation results and therefore, the density of transmitters and layout of transmitters directly influence the GDOP.

5 Conclusions and Outlook

The network topology of wMPS is analyzed and an evaluation metric of network topology based on measurement accuracy for the distributed rotary-laser measurement system is introduced. This evaluation metric, termed GDOP, derives from the mathematical model of wMPS. According to the mathematical derivation, the GDOP is a scaling factor, which amplifies the angular error of wMPS. The lower the value of GDOP, the lower the measurement error. Two significant factors of network topology affecting the GDOP, namely the layout of transmitters and the density of transmitters, are analyzed by simulations. Different layouts of transmitters cause different distributions of GDOP and a good layout can decrease the value of GDOP. More transmitters involved in measurement introduce more redundancy, and thus decrease the GDOP and measurement error. The experiments prove that GDOP is a feasible metric to quantify the effect of network topology on the measurement accuracy of wMPS. The density of the transmitter and layout of the transmitters directly influence the GDOP. With the same angular error, the lower the value of GDOP, the more accurate the wMPS.

With regard to the accuracy of this system, more detailed research into its accuracy is to be studied in the future. This may include the study of environmental vibration, and in addition, to improve the accuracy of the measurement network, a combined adjustment positioning algorithm and calibrating algorithm are needed.

Fig. 7 3-D coordinate deviation and GDOP comparison.

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