Optimization design of overall assembly tolerance for ships based on the Monte-Carlo method

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ABSTRACT

The assembly quality and assembly success rate of ships are crucial for meeting design specifications and operational performance. To improve shipbuilding efficiency and reduce construction costs, a Monte Carlo-based optimization design method for assembly tolerance of ships was proposed in the study, and then a related verification was carried out via a case. The proposed optimization design method was applied to the tolerance analysis of the assembly of marine pump equipment and its base. The analysis results were compared with the calculation results obtained through the extremum method and probability method. The results showed that the tolerance optimization design method allows for more lenient tolerance values for each component link compared to extremum method and probability method. Additionally, the assembly success rate could reach up to 99.89 % by adjusting the basic dimensions of component link A6 from the perspective of contribution rate. The optimization design method for the overall assembly tolerance of ships significantly improved the assembly success rate, highlighting its considerable potential for broader application.

Keywords: Monte Carlo method, overall design for ships, assembly tolerance optimization

1. INTRODUCTION

With the increasing demands for precise assembly of ships, the design of overall assembly tolerances has become a crucial research focus. Currently, the predominant method the predominant method to allocate assembly tolerances for a ship involves an initial determination based on regulations, standards, and the engineering experience of technicians, followed by calculations using the extremum method or probability method. These approaches, however, are often arbitrary and reliant on trial-and-error, leading to a low success rate for one-time assembly. In recent years, the Monte Carlo method has gained attention for optimizing assembly tolerances for ships. As a numerical calculation method based on probability statistics, the Monte Carlo method applies random sampling and simulation calculation to solve complex assembly tolerance issues. It allows for the comprehensive evaluation of assembly tolerances, the identification of key tolerance sources, and the proposal of corresponding optimization measures. This approach enhances the accuracy and quality of ship assembly while simultaneously reduces the need for repetitive adjustments during the assembly process, thereby improving production efficiency and lowering costs.

Many researchers[1-3] have introduced numerical models and computational equations into the tolerance analysis. Yan et al.[4] defined geometric tolerances as dimensional tolerances with a nominal value of zero and applied the Monte Carlo method to tolerance analysis, encompassing both dimensional and geometric tolerances. Kosec et al.[5] outlined the basic concepts and applications of four tolerance analysis methods, which were analyzed and compared through practical application in an open-loop assembly condition. Singh and Gulati[6,7] used the Monte Carlo method to perform tolerance analysis on both linear and nonlinear assembly cases. Additionally, they applied the Monte Carlo method to conduct zero-tolerance analysis for a dual-component linear assembly condition as early as 2019. Jing et al.[8] proposed a hybrid optimization algorithm combining the Monte Carlo method and adaptive difference method to minimize costs while ensuring assembly accuracy. The effectiveness of the algorithm was verified using an assembly case of aircraft door, with results showing that the algorithm had significant advantages in cost reduction compared to traditional methods. Antonio et al.[9] proposed a method for directly handling geometric tolerances in the process of assembly tolerance allocation, which was applicable to all optimization methods based on linear superposition models. And then two assembly cases were used to illustrate

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the convenience and high accuracy of the overall workflow in practical applications. Hallmann et al.[10] outlined the basic principles of tolerance-cost optimization methods for assembly tolerance allocation, including basic concepts, calculation formulas, and terminology. And then the author pointed out the problems and future research needs based on a review of papers on tolerance-cost optimization methods over recent decades. Li and Hou[11] established a three-dimensional assembly dimensional chain using an assembly error information transmission model. And then they analyzed the assembly dimensional chain using Monte Carlo method and compared the calculation results with traditional methods to demonstrate the effectiveness of the analysis method. Lin et al.[12] developed a computer-aided program for kinematic error distribution analysis using the Monte Carlo method, incorporating known geometric and manufacturing parameter tolerances. Subsequently, the tolerances of each parameter were optimized with the goal of minimizing manufacturing costs, and the reliability of these results was verified using the developed program. Shao et al.[13] proposed a mechanical-based tolerance analysis method to calculate elastoplastic contact behavior. The simulation results showed that this method offered high computational accuracy and efficiency, providing a feasible approach for considering the effects of elastoplastic contact behavior in tolerance analysis. Zhao et al.[14] proposed a wedge assembly tolerance analysis method based on dynamic theory. Tlijia et al.[15] introduced an assembly tolerance analysis method based on Statistical Design Theory (SDT) and finite element method (FEM), which could be implemented in a CAD environment. Liu et al.[16] considered both shape tolerances and local deformations, using a skin model shape to represent tolerances and local deformations within the model. Mu et al.[17] built a matrix model to represent deformations and compensate for the dimensional chain, verifying the proposed method through FEM. Ameta et al.[18] developed a T-Map model considering the deformations of parts during the manufacturing process, and then the distribution of assembly errors was analyzed. Goka et al.[19] established a model based on the Monte Carlo method to analyze the issues about the deformation and over-constraint. Liu et al.[20] proposed a method that accounts for environmental factors, using FEM to calculate the deformations of parts under various conditions. Zhi et al.[21] established a polyhedral model that considered machining and deformation errors to accumulate tolerances and achieve more accurate results, which were validated by an assembly case of a single-joint robotic arm. Based on this approach, Zhang et al.[22] used this method to analyze the mechanism of over-constraint. Homri et al.[23] studied tolerance analysis methods considering surface defects, and the error transmission laws under different contact conditions was analyzed.

In the study, the application of assembly tolerance design analysis methods, commonly employed in aerospace manufacturing, was extended to the shipbuilding industry. Firstly, it began with an overview of overall assembly tolerance design and allocation processes for ships, followed by detailed computational derivations of tolerance analysis methods such as the extremum method and Monte Carlo method. Next, an optimization strategy for assembly tolerance design using the Monte Carlo method was proposed. Finally, the effectiveness of the proposed approach was validated through a case involving a specific component in the ship assembly process. This study offered technical support and decision-making guidance for the shipbuilding industry, facilitating improvements in manufacturing processes and assembly quality. Additionally, the findings may serve as a reference for optimizing assembly tolerances in other fields.

2. DESIGN AND ALLOCATION FLOW OF OVERALL ASSEMBLY TOLERANCE FOR SHIPS

Tolerance design involves the control of possible assembly deviations during product design and manufacturing, and the evaluation of product assembly performance to ensure it meets assembly quality and performance requirements. The result of tolerance design affects the precision and the processing cost of ship products. In the production ship parts, shape errors would be occurred due to the complexity and variety, leading to significant error accumulation during the overall assembly of ships. The errors reduce the success rate of ship assembly and affects the structural strength and operation performance of ships. A reasonable tolerance design for ship products is crucial to ensure the success rate of ship assembly and achieve the predetermined design after assembly. In the field of ship product manufacturing, designers usually need to refine the initial results through multiple tests and tolerance analysis after the preliminary tolerance design. However, while this method ultimately meets the requirements of assembly tolerance design for ships, the factors considered in the design process are too simplistic. Therefore, the improved tolerance design may not fully account for aspects such as manufacturing or technological processes. The ideal tolerance design should not only satisfy assembly requirements but also meet the requirements of machining cost, machining method, manufacturing technology, etc. The optimal design is achieved by considering all aspects of factors.

Tolerance allocation is a process in which the tolerance of each component link is allocated economically and reasonably according to certain constraint conditions by means of tolerance design, analysis and optimization. The traditional tolerance allocation method is determined by the designer by combining the component design manual and previous engineering experience. However, for ships with complex structure, numerous functional modules and high assembly quality requirements, existing methods fall short in meeting the demands for assembly tolerance allocation. For example, if too large a tolerance range is allocated to each assembly part, the ship as a whole cannot meet the assembly requirements, but if too small tolerance range is allocated to each assembly part, it will lead to too difficult to process the parts of the ship product, the processing cost is high, and the processing efficiency is reduced.

Numerous scholars have conducted research on tolerance allocation methods, achieving notable progress. Tao et al.[24] proposed a novel cost tolerance model considering various alternative manufacturing processes' impacts on part manufacturing costs and quality losses. Their model integrates Monte Carlo simulation and a hybrid optimization algorithm involving Self-Adaptive Differential Evolution (SADE), aiming to minimize costs while ensuring high assembly precision. Ghali et al.[25] introduced an innovative tolerance allocation method based on Failure Modes, Effects, and Criticality Analysis (FMECA) and the Lagrange multiplier (LM) method. Luan et al.[26] studied the relationship between door assembly deviations and manufacturing costs, proposing a tolerance allocation model integrating manufacturing costs and Taguchi process capability index. They utilized Particle Swarm Optimization (PSO) to optimize the critical characteristics of assemblies.

Tolerance allocation is built upon assembly process design. Initially, tolerance modeling establishes mathematical models for part size tolerances and geometric tolerances, expressing tolerance information. Subsequently, initial tolerance design is performed based on functional requirements of assembly structural components. The rationality of part error and assembly deviation allocation schemes is then evaluated through theoretical calculations or computer simulation methods. Factors affecting assembly deviations and manufacturing costs are iteratively optimized to generate tolerance allocation results that meet target requirements. The design flow of overall assembly tolerance for ships is shown in Figure 1.

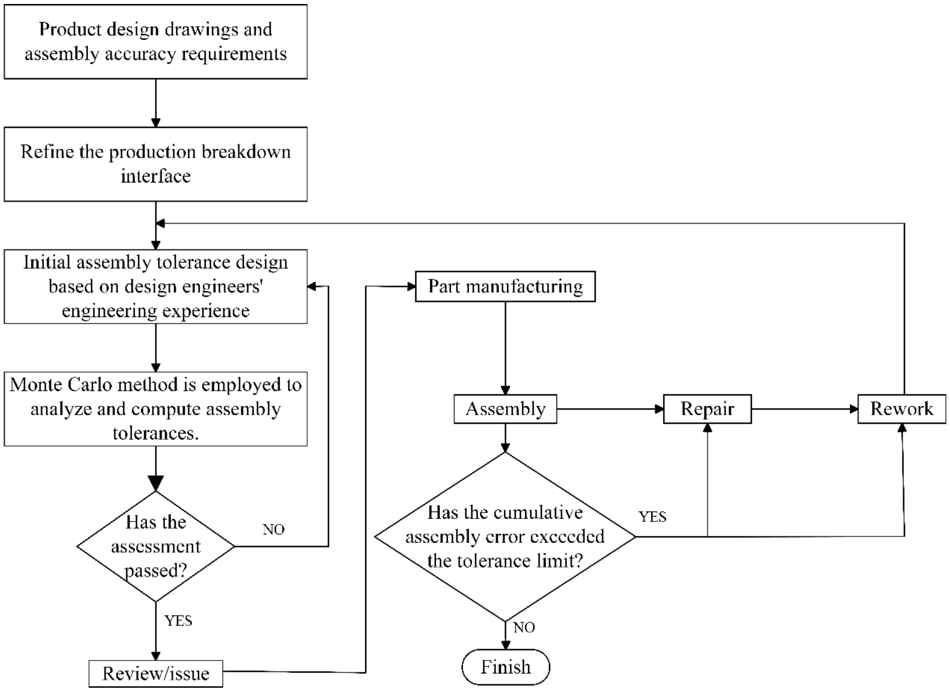

Figure 1. Design flow of overall assembly tolerance for ships.

3. ASSEMBLY TOLERANCE ANALYSIS BASED ON MONTE CARLO METHOD

3.1 Assembly tolerance analysis based on Monte Carlo method

The commonly used tolerance analysis methods are the extremum method, the probability method, and the Monte Carlo method.

3.1.1 Extremum method. The basic dimension of closing link could be expressed as:

$$A_0=\sum_{i=1}^{n}\xi_i A_i \tag{1}$$

where A_0 is the basic dimensions of closing link; A_i and ξ_i are the basic dimensions of the component link and transmission coefficient, respectively; n presents the Number of component link.

The tolerance of closing link could be expressed as:

$$\mathrm{T}_0=\sum_{i=1}^{n}\left|\xi_i\right|T_i \tag{2}$$

where T_i is the tolerances of component link; T_0 denotes the tolerances of closing link.

3.1.2 Probability method. Assuming the component link is normal distribution, the tolerances of closing link could be expressed as:

$$\mathrm{T}_0=\sqrt{\sum_{\mathrm{i}=1}^{n}\xi_{\mathrm{i}}^2 T_i^2} \tag{3}$$

When the probability distribution of component link is unknown, the empirical formula could be expressed as:

$$\mathrm{T}_0=\mathrm{H}\sqrt{\sum_{\mathrm{i}=1}^{n}\xi_{\mathrm{i}}^2 T_i^2} \tag{4}$$

where

$$H=1.8-0.8\frac{\sqrt{\sum_{i=1}^{n}\xi_i^2 T_i^2}}{\sum_{i=1}^{n}\left|\xi_i\right|T_i} \tag{5}$$

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3.1.3 Monte Carlo method

(1) The tolerance analysis steps based on the Monte Carlo method

The Monte Carlo method is a computational technique based on “random numbers”. It addresses the problem of the size and tolerance for closing link as a statistical problem of random variables, the main procedures are as follows:

a) The distribution pattern of the component link is well defined.

b) The random simulation time *N* is determined according to the computational accuracy requirements.

c) Random sampling is performed based on the distribution patterns and ranges of each dimension for the component link, yielding random numbers for each dimension $\left(A_1, A_2, \cdots A_\mathrm{n}\right)$.

d) The random numbers $\left(A_1, A_2, \cdots A_\mathrm{n}\right)$ obtained from random sampling are applied to the equation of the dimensional chain to calculate the dimension for the closing link A_0.

e) Steps (3) and (4) are repeated *N* times to obtain *N* characters of the dimension for the closing link, forming a sample.

f) The obtained samples of the dimensions for the closing link are statistically processed to determine the mean, extreme values, tolerance, and other parameters.

(2) Random simulation of dimensions for the component link

Under the condition that the basic size, upper deviation, lower deviation, and size distribution pattern of the component links are known, random numbers uniformly distributed on $(0,1)$ are generated in the stochastic simulation of the component links, and then converted into random numbers with the required distribution patterns.

Assuming T_1 and T_2 are two mutually independent random numbers uniformly distributed on $(0,1)$, then the random numbers C_1 and C_2 with a standard normal distribution $N(0,1)$ could be expressed as:

$$C_1 = \left(-2LnT_1\right)^{\frac{1}{2}} \cos\left(2\pi T_2\right) \tag{6}$$

$$C_2 = \left(-2LnT_2\right)^{\frac{1}{2}} \sin\left(2\pi T_1\right) \tag{7}$$

There is a transformation between a random number C with a standard normal distribution $N(0,1)$ and a random number y with a normal distribution $N\left(\mu, \sigma^2\right)$ is as follow:

$$y = \mu + C\sigma \tag{8}$$

The transformation of random numbers T_1 and T_2 uniformly distributed on $(0,1)$ into random numbers y_1 and y_2 with normal distribution $N\left(\mu, \sigma^2\right)$ are as follows:

$$y_1 = \mu + \sigma\left[\left(-2LnT_1\right)^{\frac{1}{2}} \cos\left(2\pi T_2\right)\right] \tag{9}$$

$$y_2 = \mu + \sigma\left[\left(-2LnT_2\right)^{\frac{1}{2}} \cos\left(2\pi T_1\right)\right] \tag{10}$$

(3) Statistical treatment of dimension samples for the closing link

After conducting *N*-time simulations of the dimensional chain for each component link, *N* subsamples $\left(X_1, X_2, \cdots, X_N\right)$ of dimensions for the closing link were obtained. These samples could be statistically processed to determine the mean, maximum, minimum, tolerance, and other parameters of the closing link.

The average $\bar{X}$ of the dimension subsamples for the closing link could be expressed as:

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N} \tag{11}$$

The maximum and minimum values $X_{\max}$ and $X_{\min}$ of the dimension subsamples for the closing link could be expressed as:

$$X_{\max} = \max\left(X_1, X_2 \cdots X_{\mathrm{n}}\right) \tag{12}$$

$$X_{\min} = \min\left(X_1, X_2 \cdots X_{\mathrm{n}}\right) \tag{13}$$

The mean value of dimensions for the closing link is equal to the average value of the subsamples, i.e., $X_{av} = \bar{X}$, the maximum and minimum values of the subsamples are the extreme values of dimensions for the closing link, respectively:

$$X_{o\max} = X_{\max}, X_{o\min} = X_{\min} \tag{14}$$

The upper deviation ES_0 and lower deviations EI_0 of dimensions for the closing link could be expressed as:

$$ES_0 = X_{o\max} - X_{av} \tag{15}$$

$$EI_0 = X_{o\min} - X_{av} \tag{16}$$

(4) Determination of sampling frequency *N*

The required number of random samples *N* for the dimension X to ensure that the proportion of X falling between the values $X_{\max}$ and $X_{\min}$ is at least p with a probability of $1-\alpha$ is:

$$Np^{N-1} - \left(N-1\right)p^N - \alpha = 0 \tag{17}$$

3.2 Contribution rate and assembly success rate

The contribution rate reflects the cumulative effect of the size tolerance information on the assembly tolerance within the dimensional chain. Dimensions with higher contribution rates exert a greater cumulative effect on controlling the target dimension, whereas those with lower contribution rates exert less influence. Equation (18) was used to calculate the contribution rate in the extremum method:

$$Contribution_{x_i} = \frac{\left|\frac{\partial U}{\partial x_i}\left(U_{x_i} - L_{x_i}\right)\right|}{U_U - L_U} \times 100\% \tag{18}$$

where x_i is the *ith* variable, U presents the assembly function (measuring object), U_{x_i} means upper limit of the tolerance for the variable x_i, L_{x_i} is lower limit of the tolerance for the variable x_i, U_U indicates upper limit of the tolerance for variable U, L_U denotes lower limit of the tolerance for variable U.

The contribution rate of the probability method is calculated as follows:

$$Contribution_{x_i} = \frac{\left(\frac{\partial U}{\partial \mathrm{x}_i}.\sigma_{x_i}\right)^2}{\sigma_U^2} \times 100\% \tag{19}$$

where σ_{x_i} is the standard deviation for the variable x_i. σ_U denotes the standard deviation for the variable U.

Assembly success rate $P(R_{\mathrm{r}})$ refers to the probability that the required dimensions fall within the given tolerance range. This is determined by finding the number N_i of dimensions among $(X_1, X_2 \cdots\cdots X_N)$ that satisfy the condition $X + \Delta_L < X_f < X + \Delta_U$ (where X is the given dimension for the closing link, Δ_L is the lower deviation specified by the designer, Δ_U is the upper deviation specified by the designer), then:

$$P(R_{\mathrm{r}}) = \frac{N_i}{N} \tag{20}$$

3.3 The research of assembly case

The dimensional chain of the installation common pump equipment and structural base assembly are shown in Figure 2.

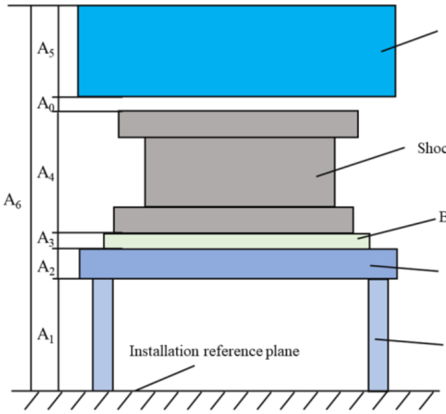

Figure 2. Installation diagram of pump equipment and relationship between assembly dimensional chain for base.

During the assembly process, $A_0 \geq 5$ mm is generally required to ensure sufficient shim clearance for precise equipment positioning, and the assembly success rate should be greater than 99%. Table 1 shows that the assembly dimensions and tolerances for base.

Table 1. Assembly dimensions and tolerances for base.

Links (Ai)	Name	Dimensions (mm)	Tolerance (mm)	Remarks
Decreasing link A_1	Cladding plate of the base	500^{+3}_{-3}	6	Design regulations
Decreasing link A_2	Panel of the base	30^{+2}_{-2}	4	Design regulations
Decreasing link A_3	Backing plate of the base	15^{+2}_{-2}	4	Design regulations
Decreasing link A_4	Shock absorber for equipment	$230^{+0.5}_{-0.5}$	1	GB-T 1804-2016
Decreasing link A_5	Machine foot height of equipment	$300^{+0.5}_{-0.5}$	1	GB-T 1804-2016
Increasing link A_6	Distance between machine foot and installation reference plane	1080^{+4}_{-4}	8	Design regulations

(1) Table 2 shows that the value calculated using the Monte Carlo method are smaller than those obtained using the extremum method, yet closely aligned with results from the probability method. It suggests that the requirements for tolerance values of each component link could be more relaxed under certain tolerance conditions for the closing link.

Table 2. Calculation results for the closing link.

	Extremum method	**Probability method—unknown component link**	**Monte Carlo method**
Closing link (mm)	5	5	5
Maximum dimensions (mm)	12	8.2	6.5
Minimum dimensions (mm)	-12	-8.2	-6.5
Assembly success rate	--	50%	53%

The contribution rate of each component link is calculated as follows:

Table 3. The assembly success rate of each component link.

Name of constituent links	**The contribution rate of extremum method**	**The contribution rate of probability method**	**The contribution rate of Monte Carlo method**
Decreasing link A_1	25.00%	26.87%	22.78%
Decreasing link A_2	16.67%	11.94%	10.89%
Decreasing link A_3	16.67%	11.94%	10.89%
Decreasing link A_4	4.17%	0.75%	6.80%
Decreasing link A_5	4.17%	0.75%	6.80%
Decreasing link A_6	33.32%	47.75%	41.84%

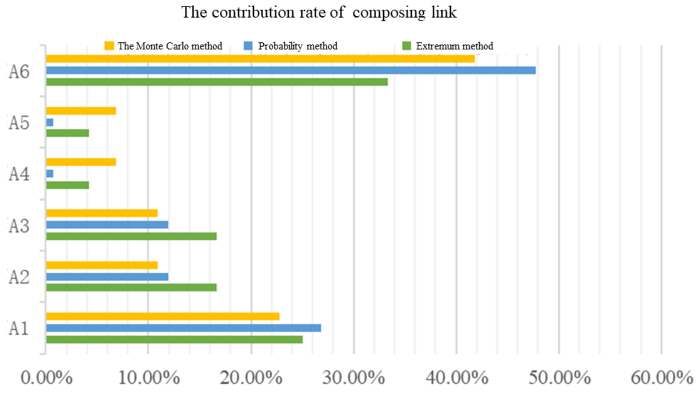

Figure 3. Contribution rate of each component link.

(2) The calculation results using the extremum method, probability method and Monte Carlo method were -12.0, -8.2 and -6.5, respectively. These results could not meet the requirements of $A_0 \geq 5$ mm, resulting in a low assembly success rate of 53%. In cases of low assembly success rates, adjustments to the basic dimensions or tolerances of the component link can be made based on their contribution rate to the closing link. As shown in Table 3 and Figure 3, the component link A6 had the most significant effect on the outcomes. To illustrate the effect of adjusting the basic dimensions of component link A6 on the assembly success rate of the closing link, the Monte Carlo method was used for the tolerance analysis. As depicted in Figure 4, the assembly success rate calculated using the Monte Carlo method reached 99.89 % when the dimension of A6 was increased to 1085 mm. The rate was significantly surpassed that of the previous study, meeting the required assembly success rate for the closing link.

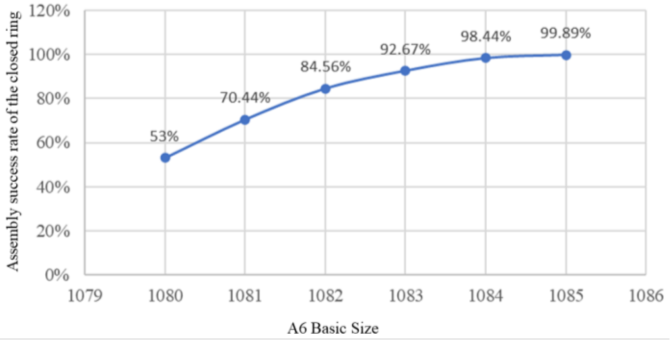

Figure 4. Tolerance optimization of the component link A6 for closing link assembly success rate.

4. CONCLUSION

An optimal design method based on the Monte Carlo method for the overall assembly tolerance of ships in this study. The proposed method was implemented for the tolerance design in the installation case of a marine pump base, with results being compared to those obtained using the extremum method and the probability method. The results indicated that the assembly success rate could reach up to 99.89% with the proposed method, significantly higher than the rate prior to adjusting the basic dimensions. It validated the effectiveness of the proposed optimization design method, offering a more scientific and applicable approach to tolerance optimization design beyond traditional assembly tolerance analysis for ships. The study offered a solution for ship assembly tolerance optimization design methods and provided valuable insights for improving overall assembly quality and success rates in shipbuilding.

REFERENCES

[1] Marziale, M. and Polini, W. A., “Review of two models for tolerance analysis of an assembly: Jacobian and Torsor,” Comput. Integr. Manuf., 24(1), 74-86 (2011).

[2] Ameta, G., Serge, S. and Giordano, M., “Comparison of spatial math models for tolerance analysis: tolerance-maps, deviation domain, and TTRS,” Journal of Computing and Information Science in Engineering, 11(2), (2011).

[3] Chen, H., Jin, S., Li, Z., et al., “A comprehensive study of three dimensional tolerance analysis methods,” Computer-Aided Design, 53, 1-13 (2014).

[4] Yan, H., Wu, X. and Yang, J., “Application of Monte Carlo method in tolerance analysis,” Procedia CIRP, 281-285 (2015).

[5] Kosec, P., Skec, S. and Miler, D. A., “Comparison of the tolerance analysis methods in the open-loop assembly,” Advances in Production Engineering & Management, 1(15), 44-56 (2020).

[6] Singh, P. K. and Gulati, V., “Tolerance analysis and yield estimation using Monte Carlo simulation—case study on linear and Nonlinear Mechanical Systems,” Sādhanā, 46(1), (2021).

[7] Singh, P. K. and Gulati, V., “Tolerance analysis of mechanical assemblies using Monte Carlo simulation—a case study,” Operations Management and Systems Engineering, (2019).

[8] Jing, T., Tian, X., Liu, X., et al., “A multiple alternative processes-based cost-tolerance optimal model for aircraft assembly,” The International Journal of Advanced Manufacturing Technology, 107(2), (2020).

[9] Antonio, A., “Allocation of geometric tolerances in one-dimensional stackup problems,” The International Journal of Advanced Manufacturing Technology, 122, (2022).

[10] Hallmann, M., Schleich, B. and Wartzack, S., “From tolerance allocation to tolerance-cost optimization: a comprehensive literature review,” The International Journal of Advanced Manufacturing Technology, 107, (2020).

[11] Lin, C., et al., “Analysis of assembly tolerance based on assembly constraint information model,”. Mathematical Problems in Engineering, (2021).

[12] Lin, K. S., Chan, K. Y. and Lee, J. J., “Kinematic error analysis and tolerance allocation of cycloidal gear reducers,” Mechanism and Machine Theory, 124, 73-91 (2018).

[13] Nan, S., Jianhua, L. and Nabil, A., “Elastic-plastic contact behaviors modeling for tolerance analysis,” Procedia CIRP, 114, (2022).

[14] Qijian, Z., Pengfei, S., Yanlong, C., et al., "Tolerance analysis of wedge assembly considering impact force," Procedia CIRP, 114, (2022).

[15] Mehdi, T., Anis, K., Borhen, L., et al., "A novel model for the tolerancing of nonrigid part assemblies in computer aided design," Journal of Computing and Information Science in Engineering, 19(4), (2019).

[16] Liu, J., Zhang, Z., Ding, X., et al., "Integrating form errors and local surface deformations into tolerance analysis based on skin model shapes and a boundary element method," Computer-Aided Design, 104, (2018).

[17] Mu, X., Sun, Q., Sun, W., et al., "3D tolerance modeling and geometric precision analysis of plane features for flexible parts," Engineering Computations, 35(7), (2018).

[18] Ameta, G., Davidson, K. J. and Shah, J. J., "Statistical tolerance analysis with T-maps for assemblies," Procedia CIRP, 75, (2018).

[19] Edoh, G., Lazhar, H., Pierre, B., et al., "Statistical tolerance analysis of over-constrained mechanical assemblies with form defects considering contact types," Journal of Computing and Information Science in Engineering, 19(2), (2019).

[20] Ting, L., Zuowei, Z., Yanlong, C., et al., "Consideration of working conditions in assembly tolerance analysis," Procedia CIRP, 75, (2018).

[21] Junnan, Z., Yanlong, C., Fan, L., et al., "Tolerance analysis of an assembly by considering part deformation," Procedia CIRP, 92, (2020).

[22] Zhang, Z., Liu, J., Answer, N., et al., "Integration of surface deformations into polytope-based tolerance analysis: application to an over-constrained mechanism," Procedia CIRP, 92, (2020).

[23] Homri, L., Goka, E., Beaurepaire, P., et al., "Framework for tolerance analysis of over-constrained mechanisms with form defects," Procedia CIRP, 92, (2020).

[24] Jing, T., Tian, X., Liu, X., et al., "A multiple alternative processes-based cost-tolerance optimal model for aircraft assembly," The International Journal of Advanced Manufacturing Technology, (2020).

[25] Maroua, G., Tlija, M. and Aifaoui, N., "Optimal tolerance allocation based on Difficulty matrix using FMECA tool," Procedia CIRP, (2018).

[26] Luan, X., Liu, H. and Liu, Y., "Research on optimization method of car door tolerance allocation based on assembly process simulation," Agricultural Equipment & Vehicle Engineering, (2018).