

receiver has a focal length of 2 m and a detector size of 1 mm. What is the correct value of dA for use in the laser range equation for this scenario?

- 1-5 A bistatic LADAR system illuminates a target that produces a Lambertian reflection. Using the simple assumptions found in the development of the range equation, what is the maximum angular separation between the transmitter and receiver that would allow a range measurement to be taken?
- 1-6 If a LADAR system receives 100 photons from the returned pulse and 10 photons of background radiation, what is the SNR of the system? Assume the coherence parameter $M = 10$ and that there is no thermal noise.
- 1-7 If a LADAR system receives 400 photons from the returned pulse and 50 photons of background radiation, what is the SNR of the system? Assume thermal noise is also present in the measurement with a standard deviation of 100 electrons, the dark current is equal to 10 nanoamps (nA), and the pulse width is 10 ns. Use a value of $M = 10$ as in the previous problem.
- 1-8 If the sun produces $1000 \text{ W/m}^2/\mu\text{m}$ of radiation on a target and the LADAR system has a square $100\text{-}\mu\text{m}$ detector pixel and a 1-m focal length, how many photons of background noise are collected during a measurement time of 10 ns? The LADAR system has a 10-cm aperture diameter, the target is Lambertian with a reflectance of 10%, and the range from the LADAR system to the target is 1 km. Also assume no transmission losses, a wavelength of $1 \mu\text{m}$ for the laser radiation, and a bandwidth of 1 nanometer for the optical rejection filter.
- 1-9 Using the computer code provided in this chapter and the LADAR system parameters contained in it, vary the detector gain by running the code for gain values of between 1 and 1000. Then plot the SNR as a function of the gain and comment on how the SNR changes as a function of the gain.
- 1-10 Using the computer code provided in this chapter and the LADAR system parameters contained in it, vary the range of the target and plot the SNR versus the range. Determine the LADAR system's effective range distance by finding the ranges for which the SNR is greater than 0.1. Assume the gain of the APD = 50. Repeat these exercises for the case where the APD gain = 1000. This problem is an example of linear-mode operation (low gain) versus Geiger-mode operation (high gain).

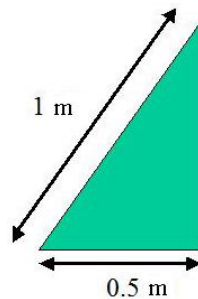


Figure 2.16 Geometry of the target for Problem 2-4.

- 2-5 Given the geometry of a LADAR target shown in Fig. 2.17, compute the target profile as a function of time. The target has a surface area of 2 m^2 .

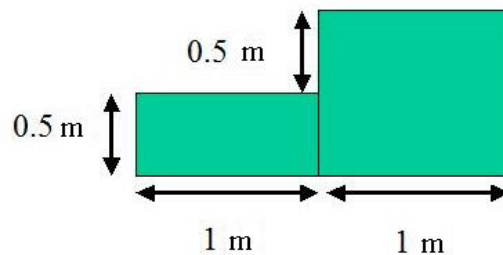


Figure 2.17 Geometry of the target for Problem 2-5.

- 2-6 A LADAR system is ranging a target that is 10 km distant. If the clock frequency is 500 MHz, how many clock cycles pass between the time when the laser is fired and when the pulse is received? If the clock frequency drifts so the actual frequency is 50 kHz higher than the nominal frequency, how many clock cycles are measured? What is the range error?
- 2-7 The LADAR system described in Example 2.1 is used to illuminate a target 100 m distant. If the speckle parameter is $M = 10$, the capacitance of the receiver circuit is 1 pF, and the background radiation is 1000 W/m^2 , what is the SNR of the waveform as a function of time? In this problem, choose the sample period to be one standard deviation of the Gaussian pulse shape.
- 2-8 Repeat Problem 2-7 using the target shown in **Figure 2.17** as the object being illuminated. Assume the target is illuminated such that both the front and back surfaces are equally visible.
- 2-9 Repeat Problem 2-7 with a negative parabolic pulse shape. Use a width parameter of the pulse equal to 2 ns.

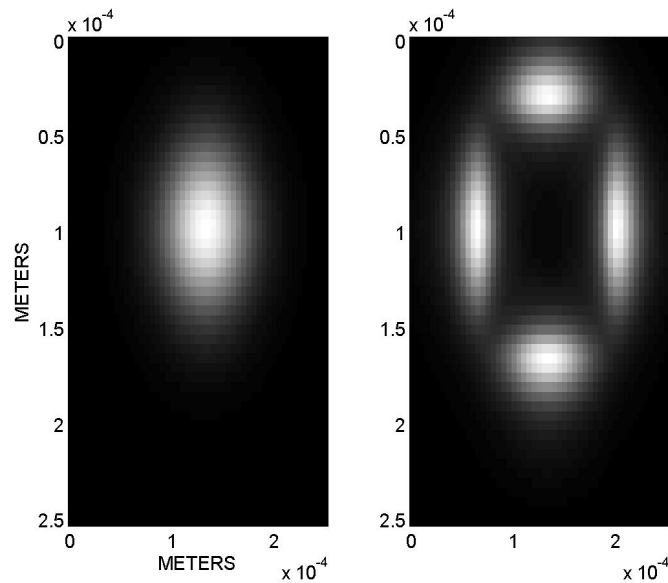


Figure 3.11 Two images of a beam returning from the target for the first pulse taken 20 ns apart. In the the first image, the beam is returning from the front surface, and in the second image, the beam is returning from the back surface of the step target.

3.5 Problems

- 3-1 Compute the size of a Gaussian beam as it propagates 1000 m to a distant target if the beam waist is 1 cm at the laser transmitter. The wavelength of the light is $1 \mu\text{m}$.
- 3-2 How large should the beam waist of a Gaussian transmit beam be to cover a 1-m-diameter circular target (the Gaussian beam at the target should be half the size of the target) if the propagation distance is 100 m? The wavelength of the light is $1 \mu\text{m}$.
- 3-3 Revisit Example 3.2 with a target that is half the size of the target at 10,000 m. Plot the detected waveform and find the relative height of the peaks generated by the front and back surfaces.
- 3-4 Compute the tilt variance for a LADAR receiver with a 1-m aperture diameter viewing through an atmosphere with a seeing parameter of 10 cm. The laser light has a wavelength of $1.06 \mu\text{m}$.
- 3-5 Compute the conditional tilt variance for the case where a LADAR receiver with a 10-cm aperture diameter is viewing an object through turbulence with a seeing parameter of 5 cm. The wind velocity across the aperture is 1 m/s, and the wavelength of the light is $1.55 \mu\text{m}$. The time between LADAR pulses is 0.1 s.
- 3-6 For the case described in problem 3.5, compute the mean of the conditional tilt in the next pulse if the current tilt is equal to zero.

The LRT, defined as Λ_2 , is produced by substituting Eq. (4.9) into the numerator of Eq. (4.7) and Eq. (4.10) into the denominator is given by

$$\Lambda_2(D) = \frac{\frac{-(D-S-B)^2}{2\sigma_1^2} - \ln(\sigma_1\sqrt{2\pi})}{\frac{-(D-B)^2}{2B} - \ln(\sqrt{2\pi B})} < 1 \text{ say } H_1, \text{ otherwise say } H_0. \quad (4.11)$$

The LRT in Eq. (4.11) features fewer computations—a number that is orders of magnitude less than the LRT shown in Eq. (4.8). Equation (4.11) uses only two natural logarithm operations, and the remaining computations are multiplication and addition operations. This is in stark contrast to gamma function calculations that are computed on every waveform data point found in Example 4.1. For this reason, the Gaussian noise approximation of the data produces a detection algorithm that is much more readily computed in real time. If the variance of the data under hypothesis H_1 is approximated as being equal to the variance under hypothesis H_0 , the LRT becomes Λ_3 :

$$\Lambda_3(D) = \frac{\frac{-(D-S-B)^2}{2B} - \ln(\sqrt{2\pi B})}{\frac{-(D-B)^2}{2B} - \ln(\sqrt{2\pi B})} < 1 \text{ say } H_1, \text{ otherwise say } H_0. \quad (4.12)$$

Λ_3 can be simplified by multiplying both sides of Eq. (4.12) by the denominator so that the inequality becomes

$$\frac{-(D-S)^2}{2B} - \ln(\sqrt{2\pi B}) < \frac{-(D-B)^2}{2B} - \ln(\sqrt{2\pi B}).$$

In this special case, the natural logarithms can be eliminated from both sides of the equation. Then both sides of the resulting equation can be multiplied by $2B$ to yield the inequality

$$-(D-S)^2 < -(D-B)^2.$$

We can further reduce the inequality by expanding the squares on both sides and removing terms that are the same. After simplification, Λ_3 becomes

$$\Lambda_3(D) = D > \frac{S}{2+B} \text{ say } H_1, \text{ otherwise say } H_0. \quad (4.13)$$



in the ability to localize the second surface. This ability to localize surfaces should translate into better estimates of the separation between surfaces, and thus, better range resolution.

4.7 Problems

- 4-1 Redo Example 4.1 using the LRT described in Eq. (4.11). Plot the new LRT versus the photocount value and determine the photocount threshold of the new LRT.
- 4-2 If a target returns 100 photons to the receiver and the background photocount value is 50, what is the threshold predicted by the LRT defined in Eq. (4.11), assuming the coherence parameter $M = 1$? What is the threshold predicted by the LRT in Eq. (4.13)?
- 4-3 Compute the probabilities of detection and false alarm for the detector described in Example 4.1. Using the threshold obtained from the LRT described in Eq. (4.13), compute the probabilities of detection and false alarm for the scenario described in Example 4.1 and compare them to the detection and false alarm probabilities obtained using the threshold from that example.
- 4-4 If the background contributes 46 photoelectrons per measurement, compute the threshold needed to obtain a probability of false alarm of 0.0001. For this threshold value, compute the probability of detection as a function of the signal strength, assuming the coherence parameter $M = 1$. Plot the probability of detection for signal levels between 1 and 100 photoelectrons.
- 4-5 Compute the number of successful detections and false alarms for the waveform detector described by Eqs. (4.16) and (4.17) by simulating 1000 waveforms from Example 4.4 when the target is both present and absent. Use those same waveforms to compute the probability of the number of successful detections and false alarms for the detector described in Eqs. (4.18) and (4.19). Which is better and why?
- 4-6 Generate a ROC curve for the scenario described in Example 4.1. Does the ROC curve depend on which LRT is used to make a decision?
- 4-7 Using the scenario in Example 4.4, compute the ROC curve for the LRT described in Eqs. (4.16) and (4.17) if the dark current is 200 nA. Also generate the ROC curve for the detector described by Eqs. (4.17) and (4.18). Plot them on the same graph and determine which waveform detector is superior based on the shape of the ROC curves.
- 4-8 Compute the sample standard deviation of the range error for the peak detector when used on the waveform data generated in Example 4.4, allowing the coherence parameter M to vary between 1 and 10. Use the peak detector with the appropriate amount of interpolation necessary to quantify the range error. Estimate the standard deviation of the range error