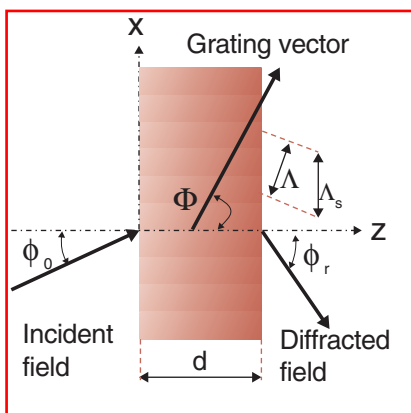


The Grating Vector

A **Bragg grating** (or, more generally, a Bragg hologram) is a volume hologram that has been recorded as an index-modulation transmittance function in an emulsion, such as silver halides, DCG, photopolymers, H-PDLC, or even photorefractives.

Although a surface-relief grating can also be a true volume Bragg grating, this is usually difficult to achieve because of the complexity of fabricating tilted, high-aspect-ratio structures.

Consider the following grating geometry and illumination, where α is the incidence angle, Φ is the diffracted angle, Λ_s is the projected period normal to the surface, and Λ is the real period of the grating.



The **grating vector** is then defined as

$$\begin{cases} \vec{K} = K(\cos(\Phi)\vec{z} + \sin(\Phi)\vec{x}) \\ K = \frac{2\pi}{\Lambda} \\ n(x, z) = n_0 + \Delta n(x, z) \end{cases}$$

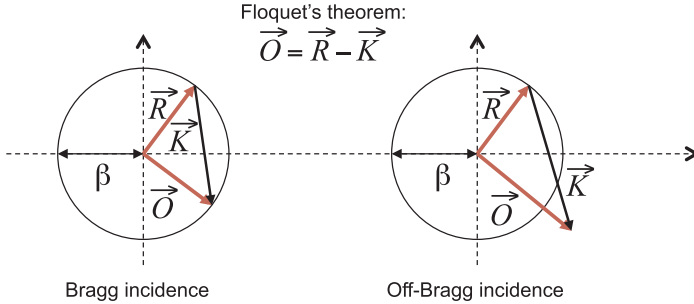
The grating-fringe slant angle, the real period, and the projected surface period can be expressed as

$$\begin{cases} \Phi = \pi/2 + (\theta_r - \theta_0)/2 \\ \Lambda = \lambda_0 / (2n_0 |\cos(\Phi - \theta_r)|) \\ \Lambda_s = \Lambda / \sin(\Phi) \end{cases}$$

New Figure

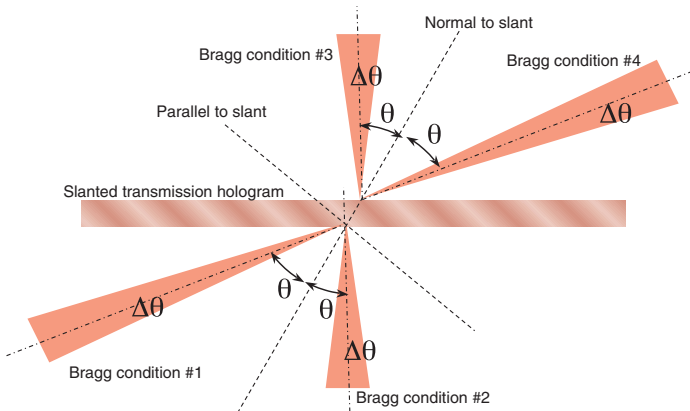
Floquet's Theorem and the Bragg Conditions

The **Bragg condition** sets the reconstruction geometry and hologram prescriptions yielding maximum efficiency. **Floquet's theorem** is expressed as a function of the grating vector \vec{K} , where \vec{O} is the object vector, and \vec{R} is the reference vector.



“On-Bragg” means that the reconstruction wavelength and angle are set to the exact optimal conditions.

The following figure shows the various Bragg conditions for a same volume grating (or volume hologram) with various wavelengths and various incidence angles.



Grating Strength and Detuning Parameter

The parameter ν_s , known as the “**grating strength**,” defines the “strength” of the physical grating as a function of the index modulation (larger is better) and thickness (thicker is better) for *s*-polarization. It is a parameter linked directly to the type of holographic recording material used:

$$\nu_s = \frac{\pi \Delta n d}{\lambda \sqrt{c_r c_s}}$$

where c_s and c_r are the obliquity factors defined as

$$\begin{cases} c_s = \cos(\alpha_i) - \left(\frac{\lambda}{n_0 \Lambda}\right) \cos(\Phi) \\ c_r = \cos(\alpha_i) \end{cases}$$

where α_i is the incidence angle, and Φ is the grating angle. The **detuning parameter** ζ (also called the dephasing parameter) is a function of the recording geometry as well as the illumination geometry and is a useful parameter describing how far one is from a Bragg condition (i.e., “on-Bragg”):

$$\zeta = \left(\frac{Kd}{2c_s}\right) \left(|\cos(\Phi - \alpha_i)| - \frac{K\lambda}{4\pi n_0}\right)$$

where K is the grating vector, d is the emulsion thickness, Φ is the diffraction angle, α_i is the incident angle, and n_0 is the base index of the emulsion.

For example, when the detuning parameter becomes zero, the Bragg condition is present (or on-Bragg). When **Floquet’s theorem** is satisfied, the detuning parameter also becomes zero. The theorem can be satisfied for multiple angle/wavelength conditions (multiple Bragg conditions).

The grating strength, obliquity factors, and the detuning factor are key parameters for the Kogelnik two-wave, coupled-mode-theory model for volume holograms (see next page).